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Explaining the Less Volatile U.S. Economy**

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Working Paper 2005-047A
<http://research.stlouisfed.org/wp/2005/2005-047.pdf>

June 2005

FEDERAL RESERVE BANK OF ST. LOUIS
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411 Locust Street
St. Louis, MO 63102

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Durable Good Inventories and the Volatility of
Production:
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First version: December 2003

This Version: November 2004

Abstract

This paper provides a simple dynamic optimization model of durable goods inventories. Closed-form solutions are derived in a general equilibrium environment with imperfect information and serially correlated shocks. The model is then applied to scrutinize some popular conjectures regarding the causes of the volatility reduction of GDP since 1984.

JEL Classification: E22, E23, E32.

Keywords: Inventory, Durable Goods, Production Volatility, Business Cycle, Demand Shock.

1 Introduction

Despite the well recognized vital role played by durable goods production and inventory investment in the business cycle, theoretical models featuring durable goods inventories are rarely available in the literature. Without such models, many important questions regarding the business cycle cannot be rigorously addressed. For example, why are durable goods production and durable goods inventory investment usually so much more volatile than that of non-durable goods? Would changes in the behavior of durable goods inventories be responsible for the dramatic reduction of output volatility since 1984? What is the likely impact of improved inventory management due to the information technology revolution on the business cycle? How would financial development change the behaviors of demand and supply of durable goods in the economy?

This paper provides a simple dynamic stochastic general equilibrium model of durable consumption goods with respect to their production and inventory accumulation. Closed-form equilibrium decision rules are characterized and derived in an environment with either perfect or imperfect information and serially correlated shocks. My model is an extension of the partial equilibrium, non-durable goods inventory models of Reagan (1982), Abel (1985) and Kahn (1987).¹ The incentive for holding inventories in these models is to insure sales against demand uncertainty when production takes time. These partial equilibrium models, however, focus only on the producers' behavior by taking demand as exogenously given. Hence these models cannot be applied directly to studying durable goods inventories, because the concept of durability is a user's measure, not a producer's measure. In this paper, demand behavior is endogenized via explicit utility maximization, making possible the studies of durable goods inventories and dynamic interactions between stochastic demand and supply.

Another independent contribution of this paper is to apply the general equilibrium model to analyze and scrutinize hypotheses about the reduction of GDP volatility since 1984, which is commonly attributed to behavioral changes in the

¹Also see Bils and Kahn (2000), Maccini and Zabel (1996).

durable goods production sectors.² Since 1984, the variance of U.S. output growth has decreased by four-fold compared to that over the post war period ending in 1983 (e.g., see McConnell and Perez-Quiros, 2000).³ This stylized fact has now received enormous amount of attention and has been scrutinized and reconfirmed by a large body of empirical literature (e.g., see Blanchard and Simon 2001, Kahn, McConnell and Perez-Quiros 2001, Kim, Nelson, and Piger 2003, Stock and Watson 2002, and Ramey and Vine 2003, among others). The consensus is that stabilization in the durable goods sector is primarily responsible for this volatility reduction. Yet exactly what has caused such a structural change in the durable goods sector is still highly controversial. Several major hypotheses are proposed in the literature to explain this apparent structural change since world-war II. Most prominently, Kahn, McConnell and Perez-Quiros (2001) argue that improvements in information technology and inventory management are the chief source of this volatility reduction. Key pieces of evidence in support of this argument are the sharp decline in the inventory-to-sales ratio since 1984 and the corresponding sharp decline in the variance of production relative to the variance of sales. Stock and Watson (2002), however, challenge this IT revolution hypothesis. Based on detailed analysis on a broad set of variables, they conclude that there is no strong evidence to show that inventory-to-sales ratio has declined since 1984 at the business-cycle frequency. Stock and Watson (2002) instead attribute most of the reduction in GDP volatility to “good luck”, that is, reductions in the variance of exogenous shocks. In a similar spirit, Ramey and Vine (2003) argue that the reduction in output fluctuations may be due to a structural change in the nature of demand shocks to consumer durables, especially automobiles. In particular, they argue that a small decrease in the volatility of sales may lead to a large decrease in the volatility of production if there are nonconvexities in production costs.

In this paper, I show that the information technology revolution can have the opposite effect on production volatility than that conjectured by Kahn et al. (2001). Namely, if the IT technology revolution amounts to improving firms’ ability to better forecast future demand by reducing noises in observing and tracking demand changes, then production should become more volatile instead of less volatile after the mid-

²General equilibrium is warranted because the structural change in the durable goods sector is large enough to affect aggregate data.

³Also see Kim and Nelson (1999).

1980s. This is because the improved ability to forecast demand changes would render inventory investment to move more closely with sales. Yet since 1984 the correlation of inventory investment to sales in the U.S. has decreased dramatically, even to a negative value, in sharp contrast with the conventional perception that this correlation is positive (see e.g., Blinder 1986 and Ramey and West 1999).⁴ On the other hand, I also show that the “good luck” hypothesis of Stock and Watson (2002) is theoretically plausible if changes in exogenous shocks stem mainly from the persistence of the shocks.⁵ In particular, the general equilibrium model implies that decreases in the persistence of demand shocks can indeed lead to decreases in the volatility of production relative to sales and this effect is much likely to be stronger for durables goods than for nondurable goods.

My general equilibrium model is related to the model of Kahn, McConnell and Perez-Quiros (2001), which also uses a general equilibrium approach to study durable goods inventories and the implications of the IT revolution for the volatility of the U.S. economy. An important difference, however, is that Kahn et al. (2001) justify the existence of inventories in equilibrium by putting inventories in the utility function, which fails to make a distinction between consumption goods and inventory goods. Such a distinction, however, is important because inventories are not the same thing as purchased goods: the former affects the market transaction price from a supply side whereas the latter does so from a demand side. In addition, my model allows for solving explicit signal extraction problems of the firm which is not able to observe demand shocks perfectly, hence my approach is more consistent with the micro-foundation of forecasting behavior of firms than is the reduced form approach of Kahn et al. (2001).⁶ These differences are responsible for the different predictions of the models regarding the implication of the information revolution on production volatility.

⁴The counter-factual implication of the IT revolution has also been conjectured by Blanchard and Simon (2001), although no formal proof is given.

⁵The “good luck” story is essentially a hypothesis rather than a fact because the so called “shocks” identified by Stock and Watson may not necessarily be truly exogenous, instead they may be black boxes reflecting endogenous movements of the economy. Hence, it is worthwhile to examine if truly exogenous shocks in a general equilibrium model can indeed replicate the volatility reduction of the U.S. economy.

⁶In Kahn et al. (2001), the IT revolution is modeled as a change in the timing of the structural shocks, whereas in my model the IT revolution enables firms to better forecast demand shocks by reducing forecasting errors.

My model is also related to the liquidity-constraint model of Deaton (1991). Inventories in my model can be re-interpreted as asset savings in Deaton's model, and the non-negativity constraint on inventories in my model can be interpreted as the borrowing constraint in Deaton's model. Thus, the incentive for holding inventories in my model is equivalent to the motive for precautionary saving in Deaton's model. A major difference, however, is that income (production) is endogenous in my model. In addition, equilibrium decision rules are derived analytically in closed forms in this paper.

The rest of this paper is organized as follows. In Sections 2 and 3, the general equilibrium model of durable goods inventories is set up, and closed-form equilibrium decision rules are derived. In Sections 4 and 5 the implication of the model for the less volatile U.S. economy is analyzed. Finally, concluding remarks are offered in Section 6.

2 The Model

Assume that the instantaneous utility function, $u(c)$, is strictly concave in the service provided by a stock of durable goods (c) and that the service flow is proportional to the stock of the goods. Also assume that production decision in period t must be made before demand in period t is known, so that firm may have an incentive to accumulate inventories to insure against demand uncertainty. A representative household chooses consumption demand for durable goods (taking price as given) to maximize life-time utility, subject to the resource constraint that discounted life-time consumption must not exceed discounted life-time labor income plus initial wealth. To simplify the analysis, physical capital is left out of the story. Hence in equilibrium household wealth is simply the stock of inventories in the economy. A representative firm chooses production and inventory investment to maximize profits (taking market prices as given). To simplify the analysis I assume a constant returns to scale production function with labor as the only production factor, which implies a linear cost function for the firm.

Applying the welfare theorems, competitive equilibrium in this model can be derived by solving a social planner's problem in which a planner chooses sequences of production, $\{y_t\}_{t=0}^{\infty}$, durable consumption goods purchases, $\{c_t - (1 - \delta)c_{t-1}\}_{t=0}^{\infty}$,

and inventory investment, $\{s_t - s_{t-1}\}_{t=0}^{\infty}$, to solve

$$\max_{\{y_t\}} E_{-1} \left\{ \max_{\{c_t, s_t\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [u(c_t, \theta_t) - ay_t] \right\} \right\}$$

subject to

$$[c_t - (1 - \delta_0 - \delta)c_{t-1}] + [s_t - (1 - \delta_0)s_{t-1}] \leq y_t \quad (1)$$

$$s_t \geq 0 \quad (2)$$

where the operator E_t denotes the expectation based on information available in period t and θ represents shocks to preferences that generate urges to consume. Assume $u''_{c\theta} > 0$, hence a positive shock to θ creates an urge to consume by increasing the marginal utility of consumption. The competitive market price for durable goods is the Lagrangian multiplier associated with the resource constraint (1). The cost of production, ay_t , is modeled as a disutility since labor is used to produce output. The linearity of the cost function is meant to keep the model tractable.

In the model, goods depreciate for two possible reasons: one is natural depreciation due to the elapse of time, measured by δ_0 ; and the other is due to wear and tear, measured by δ . Thus the rate of depreciation for purchased goods is $\delta_0 + \delta$, and the rate of depreciation for unpurchased inventories is δ_0 . To simplify notations without loss of generality, the natural depreciation rate is assumed to be zero for both purchased and unpurchased goods: $\delta_0 = 0$. The assumption that purchased durable goods depreciate faster (due to wear and tear) than unpurchased inventory goods is needed in order for inventories to exist in equilibrium. Otherwise it is optimal to hold all finished goods in the form of purchased goods instead of unpurchased inventories. Data from the automobile market shows that the value of purchased cars depreciate much faster than unpurchased cars...

Denoting λ and π as the Lagrangian multipliers associated with the resource constraint (1) and the nonnegativity constraint on inventory (2) respectively, the first order conditions with respect to $\{y_t, c_t, s_t\}$ are given by:

$$a = E_{t-1}\lambda_t \quad (3)$$

$$u'(c_t, \theta_t) = \lambda_t - \beta(1 - \delta)E_t\lambda_{t+1} \quad (4)$$

$$\lambda_t = \beta E_t \lambda_{t+1} + \pi_t \quad (5)$$

Utilizing (3), equations (4) and (5) can be simplified respectively to

$$u'(c_t, \theta_t) + \beta(1 - \delta)a = \lambda_t \quad (6)$$

$$\lambda_t = \beta a + \pi_t. \quad (7)$$

According to (6), the shadow value (competitive price) of one unit of durable goods equals its marginal utility plus the market value of the undepreciated good, $(1 - \delta)a$, measured by the production cost the agent gets to avoid paying in the next period, βa . According to (7), the value of one unit of inventory equals the discounted production cost the agent gets to avoid paying in the next period (βa), plus the shadow value of the slackness constraint (π), which is zero if the constraint does not bind. Combining (6) and (7), we have $u'(c, \theta) \geq \beta \delta a$, implying that the optimal stock of durable goods measured by its marginal utility is bounded from below by the discounted user's cost of durable goods, $\beta \delta a$.⁷

To obtain closed-form solutions, assume that the utility function is given by,

$$u(c, \theta) = \frac{(c - \theta)^{1-\alpha}}{1 - \alpha}, \quad \alpha \geq 0;$$

and the preference shock follows a stationary AR(1) process,

$$\theta_t = \rho \theta_{t-1} + \varepsilon_t,$$

where $\rho \in (0, 1)$ and $\varepsilon \sim i.i.d N(0, 1)$. To derive the decision rules of the model, consider two possibilities: the demand shock is below “normal” and the demand shock is above “normal”.⁸

Case A: If demand is below normal, then the nonnegativity constraint on inventories does not bind. Hence $\pi_t = 0$ and $s_t \geq 0$. Equation (7) implies that the shadow price of goods is constant,

$$\lambda_t = \beta a.$$

⁷Thus, the nonnegativity constraint on inventories acts like a borrowing constraint on durable consumption goods in a competitive rental market.

⁸The model is solved by the Lagrange method. See Chow (1997) for general discussions on the advantages of the Lagrange method over the Bellman dynamic programming method.

Hence equation (6) implies

$$(c_t - \theta_t)^{-\alpha} = \beta\delta a,$$

which gives the optimal consumption policy under case A,

$$c_t = \theta_t + (\beta\delta a)^{-\frac{1}{\alpha}}.$$

The resource constraint (1) then implies

$$s_t = y_t + s_{t-1} + (1 - \delta)c_{t-1} - \theta_t - (\beta\delta a)^{-\frac{1}{\alpha}}.$$

The threshold preference shock is then determined by the constraint, $s_t \geq 0$, which implies

$$\theta_t \leq y_t + s_{t-1} + (1 - \delta)c_{t-1} - (\beta\delta a)^{-\frac{1}{\alpha}}. \quad (8)$$

Case B: If demand is above normal, then the nonnegativity constraint on inventories binds. Hence $\pi_t > 0$ and $s_t = 0$. The resource constraint (1) implies that optimal consumption policy is given by

$$c_t = y_t + s_{t-1} + (1 - \delta)c_{t-1}. \quad (9)$$

To determine the optimal production policy, we can utilize equation (3). Denote $f(\cdot)$ as the probability density function of innovations in demand (ε), then

$$\begin{aligned} a &= E_{t-1}\lambda_t \\ &= \int_{-\infty}^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^{\infty} [u'(c_t, \theta_t) + \beta(1 - \delta)a] f(\varepsilon) d\varepsilon \end{aligned} \quad (10)$$

where the cutoff demand shock that determines the probability of stocking out, $z(y)$, is implied by (8). Given $\theta_t = E_{t-1}\theta_t + \varepsilon_t$, then (8) can be written as

$$\begin{aligned} \varepsilon_t &\leq y_t + s_{t-1} + (1 - \delta)c_{t-1} - (\beta\delta a)^{-\frac{1}{\alpha}} - E_{t-1}\theta_t \\ &\equiv z(y_t). \end{aligned}$$

The interpretation of (10) is straightforward. The expected value of λ is a probability distribution of two terms: $\lambda = \beta a$ if the realized demand shock is small so that

supply exceeds demand ($\pi = 0$); and $\lambda = u'(c, \theta) + \beta(1 - \delta)a$ if the realized demand shock is large so that there is a stockout ($\pi > 0$). In the latter case the optimal level of consumption is given by (9). More precisely, the left-hand side of (10) is the cost of producing one extra unit of goods today, a . The marginal benefit of having one extra unit of goods available tomorrow is given by the right-hand side of (10) with two possibilities. First, in the event of no stockout due to a low demand, the firm gets to save on the marginal cost of production by postponing production for one period. The present value of this term is βa . This event happens with probability $\int_{-\infty}^{z(y)} f(\varepsilon) d\varepsilon$. Second, in the event of a stockout due to a high demand, the firm gets to sell the product (i.e., consumption takes place). The value of this term is the marginal utility of consumption plus the present market value of the undepreciated good, $u'(c, \theta) + \beta(1 - \delta)a$, where c is determined by (9). This event happens with probability $\int_{z(y)}^{\infty} f(\varepsilon) d\varepsilon$.⁹

Clearly, the probability of stocking out, $\int_{z(y)}^{\infty} f(\varepsilon) d\varepsilon$, is determined by the level of production (y) committed one period in advance. The larger is y , the larger $z(y)$ is, hence the smaller the probability of stocking out. Since $u'(c, \theta) > \beta\delta a$ in the case of a stockout, (10) shows that an optimal cutoff point, $z(y) \in (-\infty, \infty)$, exists and it is unique given the monotonicity of the marginal utility function, $u'(c)$. This cutoff point $z(y)$ is also the optimal target level of inventories determined by the firm, which depends on the probability distribution of demand shocks and other structural parameters in general, such as $\{a, \beta, \delta\}$.

Proposition 1 *The optimal inventory target (the cutoff level of demand shock) is a constant:*

$$z(y_t) = \kappa,$$

where κ depends positively on the variance of demand shocks.

⁹Notice that the competitive goods price, λ_t , is endogenously downward sticky in general equilibrium: λ_t is constant when demand is low but increasing in θ when demand is high. This is so because, when demand is low, firms opt to hold inventories rather than to sell them at a price below marginal cost ($\lambda_t = \beta a < a$), speculating that demand may be stronger in the future. Such rational behavior attenuates downward pressure on price. When realized demand is high, on the other hand, the firm draws down its inventories until a stockout occurs and price rises to clear the market. This endogenous sticky price behavior was first noted by Reagan (1982) in a partial equilibrium model with non-durable inventories. Also see Blinder (1982) and Amihud and Mendelson (1983) for discussions on this issue.

Proof. Rewrite (10) as (utilizing equation 9):

$$\begin{aligned}
a &= \int_{-\infty}^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^{\infty} \left[(c_t - \theta_t)^{-\frac{1}{\alpha}} + \beta(1 - \delta)a \right] f(\varepsilon) d\varepsilon \\
&= \int_{-\infty}^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^{\infty} \left\{ [(y_t + s_{t-1} + (1 - \delta)c_{t-1}) - \theta_t]^{-\frac{1}{\alpha}} + \beta(1 - \delta)a \right\} f(\varepsilon) d\varepsilon \\
&= \int_{-\infty}^{z(y_t)} \beta a f(\varepsilon) d\varepsilon + \int_{z(y_t)}^{\infty} \left\{ [z(y_t) + (\beta\delta a)^{-\frac{1}{\alpha}} - \varepsilon_t]^{-\frac{1}{\alpha}} + \beta(1 - \delta)a \right\} f(\varepsilon) d\varepsilon,
\end{aligned}$$

where the last equality utilized the definition of $z(y)$. This can be simplified to:

$$(1 - \beta)a = \int_{z(y_t)}^{\infty} \left\{ [z(y_t) + (\beta\delta a)^{-\frac{1}{\alpha}} - \varepsilon_t]^{-\frac{1}{\alpha}} - \beta\delta a \right\} f(\varepsilon) d\varepsilon. \quad (11)$$

Clearly, the right-hand side of (11) is monotonically decreasing in z and it is an implicit function in the form, $g(z_t, \Omega) = 0$, where Ω is a set of parameters. Hence, the solution for $z(y)$ is unique and it must be a constant, κ , which solves $g(\kappa, \Omega) = 0$ or

$$(1 - \beta)a = \int_{\kappa}^{\infty} \left\{ [\kappa + (\beta\delta a)^{-\frac{1}{\alpha}} - \varepsilon_t]^{-\frac{1}{\alpha}} - \beta\delta a \right\} f(\varepsilon) d\varepsilon. \quad (11')$$

Clearly, κ is non-negative since the inventory target cannot be negative. Now, consider an increase in the variance of ε that preserves the mean. Since a mean preserving spread increases the weight of the tail of the distribution, the right hand side of (11') increases. Since the right hand side of (11') is decreasing in κ , κ increases when the variance of ε increases in order to maintain the equality of (11'). ■

3 Optimal Production under Imperfect Information

Given Proposition 1, the optimal level of production is then determined by

$$\kappa = y_t + s_{t-1} + (1 - \delta)c_{t-1} - (\beta\delta a)^{-\frac{1}{\alpha}} - E_{t-1}\theta_t$$

or

$$y_t + s_{t-1} + (1 - \delta)c_{t-1} = \kappa + (\beta\delta a)^{-\frac{1}{\alpha}} + E_{t-1}\theta_t.$$

Namely, optimal production is set to a level such that the total stock of goods can meet the inventory target (κ) plus the expected consumption demand, $(\beta\delta a)^{-\frac{1}{\alpha}} + E_{t-1}\theta_t$.

Since production is based on a firm's forecast of future demand, a signal extraction problem will arise if the firm is not able to observe demand shocks perfectly. Assume that what a firm can observe at the time of production decision is x , which is linked to actual demand by the relationship

$$x_t = \theta_t + u_t,$$

where u is an *i.i.d* noise term with mean zero and variance σ_u^2 and is uncorrelated with θ . The distribution of the innovations $\{\varepsilon, u\}$ and the law of motion, $\theta_{t+1} = \rho\theta_t + \varepsilon_{t+1}$, are also known to the firm. Under these circumstances, a rational firm will use the Kalman filter to forecast future demand based on the history of the observable variable, x_t . To fix notations, denote $\hat{\theta}_t$ as the forecasted demand in period t based on information in period $t - 1$:

$$\hat{\theta}_t \equiv E[\theta_t | x_{t-1}, x_{t-2}, \dots].$$

Hence, optimal production is given by

$$y_t = \kappa + (\beta\delta a)^{-\frac{1}{\alpha}} + \hat{\theta}_t - [s_{t-1} + (1 - \delta)c_{t-1}]. \quad (12)$$

By the Kalman filter, the forecasted demand follows an AR(2,1) process,

$$\hat{\theta}_t = (\rho + \gamma)\hat{\theta}_{t-1} - \rho\gamma\hat{\theta}_{t-2} + K(\varepsilon_{t-1} + u_{t-1}) + Ku_{t-2},$$

where

$$K = \frac{\rho\Sigma}{\Sigma + \sigma_u^2},$$

$$\Sigma = \frac{\sigma_\varepsilon^2 + K^2\sigma_u^2}{1 - (\rho - K)^2},$$

$$\gamma = \rho - K.$$

Notice that when there is perfect information (i.e., $u = 0$ and $\sigma_u^2 = 0$), we have $K = \rho, \gamma = 0, \Sigma = \sigma_\varepsilon^2$, and $\hat{\theta}_t = \rho\theta_{t-1}$.

Proposition 2 *The optimal decision rules for inventory holdings, durable goods purchases (sales) and production under imperfect information are given respectively by*

$$s_t = \max \left\{ 0, \kappa + \hat{\theta}_t - \theta_t \right\}$$

$$c_t - (1 - \delta)c_{t-1} = [1 - (1 - \delta)L] \left((\beta\delta a)^{-\frac{1}{\alpha}} + \min \left\{ \theta_t, \kappa + \hat{\theta}_t \right\} \right)$$

$$y_t = \hat{\theta}_t - \hat{\theta}_{t-1} + \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta \min \left\{ \theta_{t-1}, \kappa + \hat{\theta}_{t-1} \right\}.$$

where L denotes the lag operator.

Proof. Given the optimal inventory target and the implied production plan in (12), under case A discussed previously, the inequality (8) becomes $\theta_t \leq \kappa + \hat{\theta}_t$, or $\varepsilon_t \leq \kappa + \hat{\theta}_t - \rho\theta_{t-1}$. In this case, the level of inventories is given by $s_t = \kappa + \hat{\theta}_t - \theta_t$. And under case B we have $s_t = 0$. Hence inventory holdings follow the rule,

$$s_t = \begin{cases} \kappa + \hat{\theta}_t - \theta_t & \text{if } \theta_t \leq \kappa + \hat{\theta}_t \\ 0 & \text{if } \theta_t > \kappa + \hat{\theta}_t \end{cases}$$

$$= \max \left\{ 0, \kappa + \hat{\theta}_t - \theta_t \right\}.$$

Similarly, the decision rule for durable consumption goods (c_t) can be derived for case A and case B respectively as,

$$c_t = \begin{cases} \theta_t + (\beta\delta a)^{-\frac{1}{\alpha}} & \text{if } \theta_t \leq \kappa + \hat{\theta}_t \\ y_t + s_{t-1} + (1 - \delta)c_{t-1} & \text{if } \theta_t > \kappa + \hat{\theta}_t \end{cases}$$

$$= \begin{cases} (\beta\delta a)^{-\frac{1}{\alpha}} + \theta_t & \text{if } \theta_t \leq \kappa + \hat{\theta}_t \\ (\beta\delta a)^{-\frac{1}{\alpha}} + \kappa + \hat{\theta}_t & \text{if } \theta_t > \kappa + \hat{\theta}_t \end{cases}$$

$$= (\beta\delta a)^{-\frac{1}{\alpha}} + \min \left\{ \theta_t, \kappa + \hat{\theta}_t \right\}.$$

The sales of durable consumption goods are thus determined by $(1 - (1 - \delta)L)c_t$. Furthermore, since

$$y_t = \kappa + \hat{\theta}_t + (\beta\delta a)^{-\frac{1}{\alpha}} - s_{t-1} - (1 - \delta)c_{t-1},$$

substituting out s_{t-1} and c_{t-1} in y_t following the decision rules for s_t and c_t gives

$$\begin{aligned} y_t &= \begin{cases} \hat{\theta}_t - \hat{\theta}_{t-1} + \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta\theta_{t-1} & \text{if } \theta_{t-1} \leq \kappa + \hat{\theta}_{t-1} \\ \hat{\theta}_t - \hat{\theta}_{t-1} + \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta(\kappa + \hat{\theta}_{t-1}) & \text{if } \theta_{t-1} > \kappa + \hat{\theta}_{t-1} \end{cases} \\ &= \hat{\theta}_t - \hat{\theta}_{t-1} + \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta \min \left\{ \theta_{t-1}, \kappa + \hat{\theta}_{t-1} \right\}. \end{aligned}$$

■

The decision rules under perfect information can be derived easily from the decision rules under imperfect information by letting $\hat{\theta}_t = \rho\theta_{t-1}$. Notice that when goods are nondurable ($\delta = 1$) and when information is perfect ($\hat{\theta}_t = \rho\theta_{t-1}$), the decision rules in Proposition 2 become

$$\begin{aligned} s_t &= \max \{0, \kappa - \varepsilon_t\} \\ c_t &= (\beta a)^{-\frac{1}{\alpha}} + \rho\theta_{t-1} + \min \{\varepsilon_t, \kappa\} \\ y_t &= (\beta a)^{-\frac{1}{\alpha}} + \rho\theta_{t-1} + \min \{\varepsilon_{t-1}, \kappa\}. \end{aligned}$$

which are identical to those obtained by Kahn (1987) in a partial equilibrium model with exogenous demand and non-durable consumption goods, up to a constant, $(\beta a)^{-\frac{1}{\alpha}}$. This shows that Kahn's (1987) result is just a special case of the general equilibrium model.

4 IT Revolution and Output Volatility

Would the ability to better track and forecast demand due to improved information technology reduce production and inventory volatility? To answer this question, it helps to recall the variance decomposition of production:

$$\text{var}(y) = \text{var}(z) + \text{var}(i) + 2\text{cov}(z, i),$$

where z denote sales and i denotes inventory changes. This identity is derived from the identity, $y = z + i$. Hence, whether production is more volatile or less volatile than sales depends on the correlation between sales and inventory investment. If inventory investment is positively correlated with sales, then the volatility of production is

greater than that of sales, otherwise it may be less volatile than that of sales. But how much can inventory changes be correlated with sales depends on how able firms are in forecasting sales. If changes in demand come as a surprises to firms, then inventory changes will be negatively correlated with sales because firms are not able to adjust production plans immediately, hence the demand has to be satisfied solely by drawing down inventories. On the other hand, if sales are forecastable, then firms can prepare production accordingly in advance, rendering inventory investment to move more closely with sales, causing output to be more volatile relative to sales. Based on these arguments, the IT technology revolution is expected to increase output volatility rather than to decrease it.

To see this formally in the general equilibrium model, consider two extreme cases: The first case is where the variance of the noise term in the observable demand, $x = \theta + u$, is infinity ($\sigma_u^2 = \infty$), hence the signal x is not informative at all for the true size of a demand shock; the second case is where the firm has perfect information on demand shocks ($\sigma_u^2 = 0$). In the first case, the optimal forecast of θ_t based on the history of x_t is zero, $\hat{\theta}_t = E[\theta_t | x_{t-1}, x_{t-2}, \dots] = 0$, indicating that the firm's optimal production plan is no longer influenced by future demand (since it is impossible to forecast future demand); and in the second case, $\hat{\theta}_t = E[\theta_t | x_{t-1}, x_{t-2}, \dots] = \rho\theta_{t-1}$, indicating that production is heavily influenced by future demand. Thus we have the following proposition:

Proposition 3 *Output is more volatile under perfect information than under imperfect information.*

Proof. The decision rules for production under each case are given respectively by

$$y_t = \begin{cases} \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta \min \{\theta_{t-1}, \kappa\} & \text{if } \sigma_u^2 = \infty \\ \rho\theta_{t-1} - (1 - \delta)\rho\theta_{t-2} + \delta (\beta\delta a)^{-\frac{1}{\alpha}} + \delta \min \{\varepsilon_{t-1}, \kappa\} & \text{if } \sigma_u^2 = 0 \end{cases}$$

Denote $P^* = \Pr[\theta \leq \kappa]$, the variance of production under imperfect information is given by

$$\sigma_y^2 = P^* \delta^2 \sigma_\theta^2, \quad \text{if } \sigma_u^2 = \infty.$$

Denote $x_t \equiv \rho\theta_t$ and $v_t \equiv \min\{\kappa, \varepsilon_{t-1}\}$. Also denote $P \equiv \Pr[\varepsilon \leq \kappa]$. Note that the covariances, $\text{cov}(x_t, v_t) = P \times \text{cov}(x_t, \varepsilon_{t-1}) = P\rho\sigma_\varepsilon^2$ and $\text{cov}(x_{t-1}, v_t) = 0$. The decision rule for production can be rewritten as (ignoring any constants)

$$y_t = x_t - (1 - \delta)x_{t-1} + \delta v_t,$$

and the variance of production is then given by

$$\begin{aligned}\sigma_y^2 &= \sigma_x^2 + (1 - \delta)^2\sigma_x^2 - 2(1 - \delta)\text{cov}(x_t, x_{t-1}) + \delta^2\sigma_v^2 + 2\delta\text{cov}(x_t, v_t) \\ &= [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \sigma_x^2 + \delta^2\sigma_v^2 + 2\delta P\rho\sigma_\varepsilon^2.\end{aligned}$$

Since $\sigma_x^2 = \frac{\rho^2}{1 - \rho^2}\sigma_\varepsilon^2$ and $\sigma_v^2 = P\sigma_\varepsilon^2$, we have

$$\sigma_y^2 = [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \frac{\rho^2}{1 - \rho^2}\sigma_\varepsilon^2 + \delta^2 P\sigma_\varepsilon^2 + 2\delta P\rho\sigma_\varepsilon^2. \quad (13)$$

Thus,

$$\sigma_y^2 = \begin{cases} P^*\delta^2\sigma_\theta^2 & \text{if } \sigma_u^2 = \infty \\ [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \frac{\rho^2}{1 - \rho^2}\sigma_\varepsilon^2 + \delta^2 P\sigma_\varepsilon^2 + 2\delta P\rho\sigma_\varepsilon^2 & \text{if } \sigma_u^2 = 0 \end{cases}$$

where $[1 + (1 - \delta)^2 - 2(1 - \delta)\rho] > \delta^2$ since $\rho < 1$. Hence,

$$\begin{aligned}& [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \frac{\rho^2}{1 - \rho^2}\sigma_\varepsilon^2 + \delta^2 P\sigma_\varepsilon^2 + 2\delta P\rho\sigma_\varepsilon^2 \\ & > \delta^2 \frac{\rho^2}{1 - \rho^2}\sigma_\varepsilon^2 + \delta^2 P\sigma_\varepsilon^2 + 2\delta P\rho\sigma_\varepsilon^2 \\ & = \frac{\sigma_\varepsilon^2}{1 - \rho^2} [\delta^2\rho^2 + P\delta^2(1 - \rho^2) + 2P\delta\rho(1 - \rho^2)] \\ & > \frac{\sigma_\varepsilon^2}{1 - \rho^2} [P\delta^2\rho^2 + P\delta^2(1 - \rho^2) + 2P\delta\rho(1 - \rho^2)] \\ & = P\sigma_\theta^2 [\delta^2 + 2\delta\rho(1 - \rho^2)] \\ & > P\delta^2\sigma_\theta^2.\end{aligned}$$

Hence, $\sigma_y^2 = P^* \delta^2 \sigma_\theta^2$ if imperfect information and $\sigma_y^2 > P \delta^2 \sigma_\theta^2$ if perfect information. It is then obvious that output is more volatile under perfect information than under imperfect information if $P > P^*$, which is always true since¹⁰

$$P^* \equiv \Pr[\theta \leq \kappa] = \Pr[\varepsilon \leq (1 - \rho)\kappa] < \Pr[\varepsilon \leq \kappa] \equiv P.$$

■

Thus, the IT revolution does not provide explanations for the volatility reduction of the U.S. output since 1984 if it amounts essentially to improving firms' ability to observe and forecast future sales. The U.S. output may become more volatile if firms become better at tracking and forecasting future demand. This counterfactual implication of the information technology revolution has also been pointed out by Blanchard and Simon (2001). The fact, however, is that since 1984 the correlation between inventory investment and sales has decreased, instead of increasing, suggesting that, if anything, demand changes in the U.S. have become less forecastable since 1984. This is puzzling from the information revolution point of view.

5 “Good Luck” and Output Volatility

It is generally known in the literature that production is more volatile than sales for both durable and nondurable goods (e.g., see Blinder 1986, Blinder and Maccini 1991, and Ramey and West 1999). But recently the literature has also shown that since 1984, while volatilities of output and sales have both declined for both non-durable and durable goods sectors, this decline is most dramatic for the durable goods sector. In fact, it is only in the durable goods sector that the volatility ratio of output-to-sales has dropped dramatically from far above one to near or below one. For example, using quarterly data from 1953 to 2001, Kahn et al. (2001) show that,

¹⁰If $\varepsilon_j \sim N(0, \sigma)$, then $\sum_j a_j \varepsilon_j \sim N(0, \sum_j a_j \sigma)$. Hence, if we denote the standard normal *cdf* of ε ($\leq \kappa$) as $\Phi(\frac{\kappa}{\sigma})$, then

$$\Pr \left[\sum_j a_j \varepsilon_j \leq \kappa \right] = \Phi \left(\frac{\kappa}{\sum_j a_j \sigma} \right) = \Phi \left(\frac{\kappa / \sum_j a_j}{\sigma} \right) = \Pr \left[\varepsilon \leq \frac{\kappa}{\sum_j a_j} \right].$$

Therefore,

$$\Pr[\theta \leq \kappa] = \Pr \left[\sum_{j=0}^{\infty} \rho^j \varepsilon_j \leq \kappa \right] = \Pr[\varepsilon \leq (1 - \rho)\kappa].$$

before and after 1984, the variance ratio of output-to-sales for the two sub-samples has remained essentially the same (or it has even slightly increased since 1984) for non-durable goods, while that ratio for durable goods has decreased dramatically from 2.8 to 0.92. Ramey and Vine (2003) also find that the volatility ratio of output-to-sales has decreased from 2.1 to 0.6 for the automobile industry, a key industry in the durable goods sector. Thus, there are two aspects of the output volatility reduction puzzle, namely, the variance of output has decreased not only absolutely but also relative to sales, and this reduction in relative volatility is most dramatic for durable goods.

Stock and Watson (2002) argue that most of the volatility reduction in GDP since 1984 is due to “good luck”. Their analysis, which is based on a large set of macroeconomic variables, indicates that about 60-90% of the reduction in output are explained by decreases either in the variance of exogenous shock processes or in forecasting errors. A reduction in the variance of exogenous shocks can come from two sources: a decline in the variance of the innovations and a decline in the persistence of the shocks. Although Stock and Watson (2002) argue that the reduction in the variance of innovations has played a more important role, in this section I show that, from a theoretical point of view, it is a decline in the persistence of shock processes that matters for understanding the relative volatility reduction puzzle.

Proposition 4 *The variance of production decreases as the persistence of demand shocks falls, regardless the degree of durability of the goods.*

Proof. The variance of output is given by (see equation 13)

$$\sigma_y^2 = [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \frac{\rho^2}{1 - \rho^2} \sigma_\varepsilon^2 + \delta^2 P \sigma_\varepsilon^2 + 2\delta P \rho \sigma_\varepsilon^2.$$

To show that $\frac{\partial \sigma_y^2}{\partial \rho} > 0$, we need only to show that the first term is increasing in ρ , since all the other terms are increasing in ρ . Differentiating the first term with respect to ρ gives

$$-2(1 - \delta) \frac{\rho^2}{1 - \rho^2} + [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \frac{2\rho}{(1 - \rho^2)^2},$$

which is positive if and only if

$$(1 - \delta)\rho(1 - \rho^2) < [1 + (1 - \delta)^2 - 2(1 - \delta)\rho],$$

which can be simplified to

$$\delta^2 + a(1 - \delta) > 0,$$

where $a \equiv (1 - \rho)[2 - \rho(1 + \rho)]$. Since $a > 0$ for $\rho \in (0, 1)$, the above inequality always holds for any value of $\delta \in [0, 1]$. ■

The intuition behind Proposition (4) is that as ρ decreases, demand shocks become less forecastable, hence production is less responsive to changes in expected future demand. Under this circumstance, optimal production is dictated more by the inventory target (κ) than by expected demand, leading to less volatile output.

Proposition 5 *The relative volatility of production to sales can be either larger than one or less than one, depending on the persistence of the shocks. However, this volatility ratio decreases surely as the persistence of demand shocks falls only if goods are sufficiently durable. When goods are non-durable, this volatility ratio may not necessarily decrease as ρ falls.*

Proof. Denote $P \equiv \Pr[\varepsilon \leq \kappa]$, and $q_t \equiv c_t - (1 - \delta)c_{t-1}$, the variance of durable goods sales is then given by

$$\sigma_q^2 = [P + (1 - P)\rho^2] [1 + (1 - \delta)^2 - 2(1 - \delta)\rho] \sigma_\theta^2.$$

Using equation (13), simplification gives

$$\sigma_y^2 - \sigma_q^2 = 2P[\rho + \delta - 1] \sigma_\varepsilon^2.$$

Hence, variability of production can be larger than that of sales if $\rho + \delta > 1$, or it can be smaller than the variability of sales if $\rho + \delta < 1$. The variance ratio of output-to-sales is given by

$$\frac{\sigma_y^2}{\sigma_q^2} = 1 + \frac{2P[\rho + (\delta - 1)](1 - \rho^2)}{[P + (1 - P)\rho^2][1 + (1 - \delta)^2 - 2(1 - \delta)\rho]}. \quad (14)$$

Notice that this ratio is independent of the variance of the innovations in the demand shocks (σ_ε^2), but it is a function of the persistence parameter of the shocks (ρ). Since

it is difficult to evaluate the partial derivatives of this equation, in order to show the effect of a change in the persistence of shocks on the variance ratio, we can consider two extreme cases: $\delta = 0$ and $\delta = 1$. We need to show that: 1) when $\delta = 0$ (i.e., goods are extremely durable), the right-hand side of (14) increases with ρ , suggesting that a decrease in the persistence of shocks leads to a decrease in the variance ratio; 2) when $\delta = 1$ (i.e., goods are non-durable), the right-hand side of (14) is hump-shaped, and it is decreasing in ρ for $\rho \geq \bar{\rho}$ and increasing in ρ for $\rho < \bar{\rho}$, suggesting that the variance ratio will not necessarily decrease as ρ falls. Let $\delta = 0$, we have

$$\frac{\sigma_y^2}{\sigma_q^2} = \frac{\rho^2}{P + (1 - P)\rho^2}.$$

The derivative of the right-hand side with respect to ρ is positive. Let $\delta = 1$, we have

$$\frac{\sigma_y^2}{\sigma_q^2} = 1 + \frac{2P\rho(1 - \rho^2)}{[P + (1 - P)\rho^2]},$$

which is obviously hump-shaped with a maximum at $\bar{\rho} \in (0, 1)$, since $\sigma_y^2/\sigma_q^2 = 1$ if $\rho \in \{0, 1\}$ and $\sigma_y^2/\sigma_q^2 > 1$ if ρ is not at the boundary of $[0, 1]$.¹¹ ■

The above analysis shows that a change in the variance of innovations (ε) originating from preferences has no effect on the ratio of output volatility to that of sales, but a change in the persistence of the shocks could have a large effect on the volatility ratio. In particular, it is shown that a decrease in the persistence of demand shocks can always lead to a decrease in the absolute volatility of output for both durable and non-durable goods; however, a decrease in the persistence of demand shocks will unambiguously lead to a decrease in the variance ratio of output-to-sales only if goods are sufficiently durable. These implications suggest that a change in the nature of exogenous shocks – the “good luck” hypothesis – is a theoretically plausible story for the less volatile U.S. economy since 1984, although it is the change in the persistence of shocks, not in the variance of forecasting errors or innovations, that matters (in theory).¹²

¹¹For example, suppose $P = 0.5$, then $\bar{\rho} = 0.48587$ and $\sigma_y^2/\sigma_q^2 = 1.6$.

¹²The result here is not necessarily inconsistent with Stock and Watson (2002). Stock and Watson (2002) use endogenous variables to identify the changes in innovations and persistence of shocks. But truly exogenous shocks are not observable and a reduction in the persistence of truly exogenous shocks may not necessarily imply a significant reduction in the persistence of the observable “shocks” identified by Stock and Watson.

6 Concluding Remarks

Durable goods production and inventory investment have played vital roles in business cycles. Yet theoretical models of durable goods inventories are rarely available in the literature. Thus many important empirical issues relating to the business cycle cannot be rigorously addressed by theory. This paper provides a simple dynamic general equilibrium model of durable goods inventories and applies the model to analyze a prominent feature of the post war U.S. economy.

The fact that the U.S. economy has become less volatile since the early 1980's has sparked immense interest in searching for its causes. The empirical evidence strongly suggests that stabilization in the durable goods sector since the early 1980's holds the key for the decline in GDP volatility. This structural change could be technology driven (as advocated by Kahn, McConnell and Perez-Quiros, 2001), or it could be demand shock driven or "good luck". A crucial question, which each of these theories must answer, is why this structural change is more prominent for the durable goods sector than for the nondurable goods sector?

The general equilibrium model developed in this paper is applied to differentiate and sharpen the predictions of existing hypotheses regarding the less volatile U.S. economy. It is shown that the information revolution story may yield counter-factual predictions if we assume that the IT revolution amounts essentially to enhancing firms' abilities to observe and forecast demand. The good-luck story, on the other hand, is shown to be theoretically more coherent.¹³ While these implications of the simple general equilibrium model are interesting, further work is clearly needed in order to validate the "good luck" hypothesis, especially to refine the definition of demand shocks. In the model, changes in demand are caused by shocks to preferences. Such shocks are not observable, hence cannot be directly measured. A natural next step in this line of research is to find a way to determine whether the preference shock story is observationally equivalent to changes in other fundamentals in the economy. It is possible, for example, that the assumed change in the preference shock process since 1984 reflect households' responses to a changing macro economic environment, such as changes in the government monetary policy or in the financial system that

¹³My analysis, however, does not exclude that there are other channels of the IT revolution through which the volatility of production may be adversely affected.

has eased credit availability or borrowing constraints (e.g., see Blanchard and Simon, 2001). In the simple general equilibrium model, endogenous responses from demand to environment changes may have been captured instead as exogenous preference shifts. This possibility is worth further investigation (see Antinolfi and Wen, 2003). The general equilibrium framework provided in this paper, however, may serve as a natural vehicle for carrying out further analysis on lines like this.

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