



WORKING PAPER SERIES

Worker Turnover, Industry Localization, and Producer Size

Christopher H. Wheeler

Working Paper 2004-021A  
<http://research.stlouisfed.org/wp/2004/2004-021.pdf>

September 2004

FEDERAL RESERVE BANK OF ST. LOUIS  
Research Division  
411 Locust Street  
St. Louis, MO 63102

---

The views expressed are those of the individual authors and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors.

Federal Reserve Bank of St. Louis Working Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Federal Reserve Bank of St. Louis Working Papers (other than an acknowledgment that the writer has had access to unpublished material) should be cleared with the author or authors.

Photo courtesy of The Gateway Arch, St. Louis, MO. [www.gatewayarch.com](http://www.gatewayarch.com)

# Worker Turnover, Industry Localization, and Producer Size

Christopher H. Wheeler\*

Research Division

Federal Reserve Bank of St. Louis

411 Locust Street

St. Louis, MO 63102

Christopher.H.Wheeler@stls.frb.org

Tel.: 314-444-8566

Fax: 314-444-8731

August 4, 2004

## Abstract

Empirically, large employers have been shown to devote greater resources to filling vacancies than small employers. Following this evidence, this paper offers a theory of producer size based on labor market search, whereby a key factor in the determination of a producer's total employment is the ease with which workers can be found to fill jobs that are, periodically, vacated. Since the geographic localization of industry has long been conjectured to facilitate the search process, the model provides an explanation for the observed positive association between average producer size and the magnitude of an industry's presence within local labor markets.

---

\*I wish to thank an anonymous referee and seminar participants at the University of Georgia for their comments. I am also grateful to Stefani Smith whose suggestions greatly improved the paper. Any errors, of course, are my own. The views expressed herein do not necessarily represent those of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

# 1 Introduction

Production establishments vary considerably in terms of the number of workers they employ. In the United States, for instance, manufacturing plants are classified by size categories ranging from fewer than 10 employees to over 5000. What accounts for this variation? Why are some economies populated by relatively large production establishments whereas others contain mostly small ones?

The answers, I believe, may offer valuable insights into a number of important economic issues. For example, larger plants have been shown to pay higher wages (Brown and Medoff 1989 and Troske 1999) and enhance worker productivity (Idson and Oi 1999). Thus, an economy's aggregate labor earnings and efficiency may be tied to the extent to which it organizes its workers into large establishments rather than small ones. Moreover, an economy's firm size distribution may also help to account for a number of additional outcomes, including the degree to which it invests in research and development (Cohen and Levin 1989), the nature of its short run employment fluctuations (Evans 1987, Dunne et al. 1989, Audretsch and Mahmood 1995, and Davis et al. 1996), and its path of long run growth (Fukuyama 1995 and Kumar et al. 1999).

Unsurprisingly, then, the literature studying the determinants of producer size is extensive. Indeed, for decades, theories of the firm have suggested a variety of technological and institutional factors, including the distribution of skill (e.g. Lucas 1978, Rosen 1982, and Kremer 1993), the effectiveness of an economy's judicial and financial systems (e.g. Kumar et al. 1999) and its regard for property rights (e.g. Grossman and Hart 1986 and Hart and Moore 1990), as key elements underlying a producer's overall scale.<sup>1</sup>

---

<sup>1</sup>Kumar et al. provide a recent survey of this work as well as some empirical evidence on several of the

Notwithstanding these previous contributions, this paper suggests that producer size may also be influenced by an element that, surprisingly, has been largely overlooked: the costs associated with labor market search. That is, the number of employees that a firm chooses to hire may depend critically on the expected cost of identifying, recruiting, training, and replacing those workers periodically. To be sure, researchers have long noted that firms face substantial fixed hiring and training costs (e.g. Oi 1962), which may help to explain patterns of worker turnover with respect to wages or skill. This paper argues that these costs may also influence the overall size that firms achieve.

Two pieces of empirical evidence provide the basic motivation for this idea. First, the quantity of resources devoted to filling a vacancy tends to rise with producer size. Barron et al. (1985), for instance, show that large production establishments spend more hours screening, interviewing, and recruiting per worker, interviewing, on average, a larger number of workers per position than small ones. The specific figures they report are reproduced in Table 1.

Second, employment turnover is significantly higher in terms of absolute numbers of workers coming and going among large employers. Notably, Davis et al. find that over the period 1973 to 1988, manufacturing establishments with 1000 to 2499 employees experienced an average annual gross job reallocation rate (the sum of job creation and destruction expressed as a fraction of total employment) of 13.5 percent. For plant sizes 2500 to 4999 and 5000 or more employees, gross job reallocation rates averaged 13.6 percent and 10.9 percent, respectively.

While small manufacturing plants experienced higher reallocation rates (those with 0

---

theories.

to 19, 20 to 49, and 50 to 99 employees experienced rates of 42 percent, 28.6 percent, and 25.6 percent respectively) this evidence indicates that large plants must search for a considerably larger absolute number of workers in a typical year than small ones. Hence, large establishments need to be able to find and hire greater numbers of workers simply as a result of the natural turnover they face each year.

When combined, these two sets of results suggest that worker turnover and labor market search become an increasingly significant concern as employers hire greater numbers of employees. One should expect, therefore, that producers wishing to employ large numbers of workers will tend to locate in markets with lower search costs. Alternatively, to the extent that there are economies of scale in production, firms who find themselves in environments in which search is easy should tend toward larger equilibrium sizes. Either way, the same basic conclusion emerges: markets that facilitate labor market search will tend to organize their economic activity around larger production establishments.

Conceptually, of course, the notion that producer scale may be influenced by the costs of managing groups of workers is not new. Most famously, Coase (1937) suggested that the reason why firms exist at all is to economize on the costs associated with coordinating transactions (e.g. writing contracts), which tend to be lower between agents within firms than between agents in a decentralized market. Concerning the eventual size that a firm achieves, Coase (p. 396) noted that “a firm will tend to be larger . . . the less the costs of organizing and the slower these costs rise with an increase in the transactions organized.”

While much of the literature has characterized these organization costs as those stemming from managing or monitoring groups of workers (e.g. Williamson 1967 and Rosen 1982), the evidence just surveyed indicates that this concept may be extended to include

the cost of maintaining a workforce that turns over periodically. With worker turnover, firms must engage in costly search to replace those who leave for one reason or another. Hence, there may be a significant labor search aspect to the issue of producer size. This paper attempts to formalize this idea.

The formal analysis appears in the next section, which develops the model and derives the basic theoretical results regarding the determinants of employer size. Section 3 then offers a brief discussion of the model's empirical implications. In particular, because the geographic concentration of industry (i.e. 'localization') has long been held to facilitate a producer's search for workers (as well as a worker's search for employers), the model implies that firms situated in heavily localized areas ought to be larger. Casual evidence seems to bear this implication out.

## 2 A Model of Producer Size

### 2.1 Baseline Specification

Consider an economy comprised of a single producer (or firm) and a continuum of workers.<sup>2</sup> The producer is endowed with an exogenously determined quantity of physical capital,  $k$ , and must hire workers to use this capital in the production of output. Each worker, in turn, is characterized by a level of skill,  $q$ , equal to one of  $T$  possible values:  $q_1, q_2, \dots, q_T$ , where  $q_1 \geq q_2 \geq \dots \geq q_T > 0$ . Let the proportion of the total worker population with skill level  $q_t$  be denoted by  $\mu_t, t = 1, 2, \dots, T$  where  $\sum_{t=1}^T \mu_t = 1$ .

The hiring process, I assume, takes the following simple form. For each of an endoge-

---

<sup>2</sup>By assuming a continuum, I am restricting the worker skill distribution to be stationary in the presence of discrete numbers of draws.

nously determined number of vacancies, the producer takes random draws from the labor force until an ‘acceptable’ worker (that is, one with a satisfactory skill level,  $q$ ) is found. The firm then fills the vacancy by hiring this worker.<sup>3</sup>

The collection of workers that a firm ends up hiring can be represented by a profile  $(N_1, N_2, \dots, N_T)$ , where each  $N_t \geq 0$  denotes the number of workers of skill level  $q_t$  that the firm employs. Given such a profile, the primary object of interest in this model, the producer’s size, will be summarized by the parameter  $N = \sum_{t=1}^T N_t$ .

I specify the firm’s production technology as a Cobb-Douglas function of two elements: (i) the firm’s stock of physical capital,  $k$ , and (ii) a CES aggregate of the skills of the workers hired. Hence, the production generated by the firm with worker profile  $(N_1, N_2, \dots, N_T)$  in a given period of time follows as

$$Y = k^\alpha \left( \sum_{t=1}^T N_t q_t^\rho \right)^{\frac{1-\alpha}{\rho}} \quad (1)$$

where  $0 < \alpha, \rho < 1$ . For simplicity, suppose that  $\rho = 1 - \alpha$  so that equation (1) can be re-written as

$$Y = k^\alpha \left( \sum_{t=1}^T N_t q_t^{1-\alpha} \right) \quad (2)$$

This output is then divided between the producer and its workers according to an  $\alpha$ ,  $(1 - \alpha)$

---

<sup>3</sup>This process, of course, implies that workers accept all job offers they receive. Such an assumption, I hold, is not completely unreasonable. Barron et al. (p. 50), for example, note that, in their data, “approximately 90 percent of jobs are filled by a single job offer.”

split. Thus, the firm pays its employees  $(1 - \alpha)Y$  in aggregate and keeps the remainder,  $\alpha Y$ , for itself. This particular division can be justified by assuming that workers receive their marginal products,  $(1 - \alpha)k^\alpha q_t^{-\alpha}$ , so that the firm's 'gross payoff' (i.e. its share of the output produced) in each period is

$$Y - (1 - \alpha)k^\alpha \left( \sum_{t=1}^T N_t q_t^{-\alpha} q_t \right) = \alpha Y.$$

Notice, given the specification of the production function, the firm's gross payoff strictly increases in the number of workers employed: adding an additional worker always increases  $\alpha Y$ . Of course, the amount by which an additional worker increases  $\alpha Y$  depends on the skill level of that worker: higher  $q$  workers add more than lower  $q$  workers. So, even though high-skill workers earn higher wages in this framework,  $(1 - \alpha)k^\alpha q_t^{-\alpha} q_t = (1 - \alpha)k^\alpha q_t^{1-\alpha}$  strictly increases in  $q_t$ , the producer's gross return to employing another worker strictly increases in the worker's skill level.

Maintaining a workforce of  $N$  employees, however, is costly in two respects. First, I assume that there is a coordination cost given by  $N^\phi$  units of output per time period, where  $\phi > 1$ . Intuitively, we can think of this term as representing the idea that, as the number of workers at the same firm grows, it becomes increasingly costly to monitor or manage their activities.<sup>4</sup>

---

<sup>4</sup>This term also ensures that the firm's problem has a well-defined solution in the sense that, without it, the optimal number of workers to hire would be infinite. For ease, I assume that this coordination term is independent of the skills of the workers hired. While one might argue that more highly skilled workers are, in some sense, easier to coordinate than less skilled workers, high-skill workers may also have more complex tasks to complete (e.g. Kremer) that should increase the costs of coordination. Because the net effect is not, a priori, clear, I utilize the expression above.

Second, the firm faces worker turnover. With a constant probability  $\delta$ , a hired worker will leave the firm in a given time period. Operating with a total of  $N$  workers, therefore, will also require the producer to replace, on average, a total of  $\delta N$  workers in each period. To capture the costs associated with this process, I assume that a firm replaces a worker by randomly drawing potential employees from the labor force at a cost of  $C$  units of output per draw until an acceptable worker is found.<sup>5</sup>

On average, then, the cost associated with turnover can be expressed as the product of two terms: the expected number of replacements per period and the expected cost per replacement. The first piece is simply  $\delta N$ . The second can be derived by noting that, in a Bernoulli process with probability  $p$  of ‘success,’ the expected number trials before the first success is  $\frac{1}{p}$  (see, for example, Cinlar 1975, p. 57). Thus, since the probability that a random draw from the labor force will yield an acceptable worker is  $\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)$  where  $1(\cdot)$  is an indicator function taking a value of 1 if  $q_t$  is contained in the set of acceptable skill levels  $\Omega$  and 0 otherwise, the expected cost to the producer of replacing one vacated position is  $\frac{C}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)}$ . The expected replacement cost per period, then, follows as

$$\frac{C\delta N}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)}. \quad (3)$$

---

<sup>5</sup>Although one could certainly argue that  $C$  may, itself, be a decreasing function of the turnover probability  $\delta$  (i.e. greater turnover implies a larger set of workers seeking employment, therefore, a lower cost of drawing one worker), I treat the two parameters  $C$  and  $\delta$  as separate. Doing so, I hold, is innocuous in terms of the inferences drawn from the model since the comparative-static results derived below with respect to changes in  $C$  would be similar to those derived from changes in the product  $C\delta$ . More importantly, as I suggest in Section 3, aspects of a labor market (e.g. its overall size, density, or rate of growth) may influence  $C$  independently of  $\delta$  permitting for an evaluation of the effects of a change in  $C$  holding  $\delta$  constant.

Ultimately, what the firm seeks to maximize is the average level of profit (i.e. the expected value of its gross payoff,  $\alpha Y$ , net of both coordination and expected turnover costs) that it receives over time.<sup>6</sup> Realizing that more than one acceptable level of worker skill means that the exact composition of its employed workforce will (likely) vary from one time period to another due to turnover, the expected value of the producer's gross payoff,  $E(\alpha Y)$ , can be calculated as follows. Note first that, from (2), when  $N$  workers (whose identities are indexed by  $j$ ) are hired,  $\alpha Y$  can be written as  $\sum_{j=1}^N \alpha k^\alpha h_j^{1-\alpha}$  where  $h_j$  denotes the skill level of worker  $j$ . The expected value of this term, then, can be expressed as

$$E(\alpha Y) = \sum_{j=1}^N \alpha k^\alpha E(h_j^{1-\alpha} | h_j \in \Omega) = \alpha k^\alpha N E(h_j^{1-\alpha} | h_j \in \Omega) \quad (4)$$

where, again,  $\Omega$  is the set of skill levels the firm deems acceptable. Because openings are filled from sets of random draws from a population with (known) skill distribution  $(\mu_1, \mu_2, \dots, \mu_T)$ ,  $E(\alpha Y)$  can be written more explicitly as

$$E(\alpha Y) = \alpha k^\alpha N \frac{\sum_{t=1}^T q_t^{1-\alpha} \mu_t \cdot 1(q_t \in \Omega)}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)}. \quad (5)$$

Combining this expression with those for the coordination and expected turnover costs yields the following equation for the producer's expected profits,  $\pi$ :

---

<sup>6</sup>By assumption, then, the firm is not concerned with the costs associated with finding its first  $N$  employees, only the costs of coordinating and maintaining  $N$  workers over time.

$$\pi = \alpha k^\alpha N \frac{\sum_{t=1}^T q_t^{1-\alpha} \mu_t \cdot 1(q_t \in \Omega)}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)} - \frac{C\delta N}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)} - N^\phi. \quad (6)$$

The first result that I establish is that the maximization of (6) involves the producer's use of a cutoff strategy when selecting workers from the labor force.

**Proposition 1:** The set of workers that the producer finds acceptable satisfies a cutoff property whereby if any skill level  $q^*$  is acceptable, so is any skill level  $q > q^*$ .

*Proof:* Suppose that a producer of given size  $N$  is willing to hire a worker of skill  $q^*$ . Now consider the effect of adding any  $q > q^*$  to the set of acceptable skill levels,  $\Omega$ . First, the inclusion  $q > q^*$  will strictly increase the expected gross payoff,  $E(\alpha Y)$ , by increasing the expected value of a hired worker's skill raised to the power  $(1 - \alpha)$ . Second, it will decrease the expected replacement cost  $\frac{C\delta N}{\sum_{t=1}^T \mu_t \cdot 1(q_t \in \Omega)}$  by decreasing the expected number of draws required to find the first acceptable worker. Thus, because the overall effect is to increase expected profits, if  $q^* \in \Omega$ , then  $q \in \Omega$  for any  $q > q^*$ .

At the same time, including a lower skill level  $q < q^*$  in  $\Omega$  will not always be worthwhile. Although doing so decreases the expected replacement cost, it also decreases the expected gross payoff. Hence, the set of acceptable workers satisfies a cutoff property. ■

The firm's problem, then, consists of two tasks: (i) setting a reservation level of worker skill, and (ii) determining the number of employees that maximizes expected profits under

this cutoff. I assume the firm solves this problem in the following two stages. First, it maximizes expected profits with respect to the number of workers, taking the cutoff skill level as given. Second, after having found an optimal number of workers for each of the  $T$  possible cutoff levels,  $N(q_t)$ ,  $t = 1, 2, \dots, T$ , the producer maximizes its expected profits over  $N(q_t)$ .

Solving such a problem is straightforward. Take the firm's cutoff, say equal to  $q_p$ , as given. Expected profits under such a cutoff,  $\pi(q_p)$ , are simply

$$\pi(q_p) = \alpha k^\alpha N \left( \sum_{j=1}^p \frac{q_j^{1-\alpha} \mu_j}{\mu_1 + \mu_2 + \dots + \mu_p} \right) - \frac{C\delta N}{\mu_1 + \mu_2 + \dots + \mu_p} - N^\phi. \quad (7)$$

Maximization of (7) with respect to the number of workers,  $N$ , implies the following optimal size

$$N(q_p) = \left( \frac{\alpha k^\alpha (q_1^{1-\alpha} \mu_1 + \dots + q_p^{1-\alpha} \mu_p) - C\delta}{\phi(\mu_1 + \dots + \mu_p)} \right)^{\frac{1}{\phi-1}}. \quad (8)$$

The selection of a cutoff skill level then follows from the calculation of the optimal  $N$  for each of the  $T$  possible values of  $q_p$ , substituting the resulting sizes into the expression for expected profits, and finding the largest value.

## 2.2 The Case of Two Skill Types

To simplify matters at this point, let there be only two skill types, 1 and 2, with  $q_1 > q_2$ . The corresponding proportions of the labor force accounted for by type 1 (i.e. 'high-skill') and 2 (i.e. 'low-skill') workers can then be reduced to  $\mu_1 = \mu$  and  $\mu_2 = 1 - \mu$ . Following

the analysis above, a producer's optimal size is

$$N(q_1) = \left( \frac{\alpha k^\alpha q_1^{1-\alpha} \mu - C\delta}{\phi \mu} \right)^{\frac{1}{\phi-1}} \quad (9)$$

for a cutoff of  $q_1$  and

$$N(q_2) = \left( \frac{\alpha k^\alpha (q_1^{1-\alpha} \mu + q_2^{1-\alpha} (1-\mu)) - C\delta}{\phi} \right)^{\frac{1}{\phi-1}} \quad (10)$$

for a cutoff of  $q_2$ . To ensure that both optimal producer sizes are positive, I assume that the model's parameters satisfy the following condition which, in essence, simply restricts the search cost parameter  $C$  to be sufficiently small:

$$\alpha k^\alpha q_1^{1-\alpha} \mu - C\delta > 0.$$

The following result characterizing the optimal size chosen by a producer follows directly from expressions (9) and (10).

**Proposition 2:** For a given cutoff skill level, (i) an increase in the proportion of type 1 (i.e. high-skill) workers,  $\mu$ , and/or (ii) a decrease in the search cost parameter,  $C$ , will increase the number of workers that the producer optimally hires.<sup>7</sup>

---

<sup>7</sup>Notice, optimal firm size also increases in the physical capital of the producer,  $k$ , which, if interpreted as the 'ability' of the producer, is similar to the implications of Lucas (1978), Rosen (1982), and Waldman (1984).

*Proof:* This result can be derived by differentiating (9) and (10). Because  $\phi > 1$ , either an increase in  $\mu$  or a decrease in  $C$  increases  $N(q_1)$  and  $N(q_2)$ . ■

This result merely establishes that the number of employees that a producer will optimally hire is an increasing function of the fraction of high-skill workers relative to the fraction of low-skill workers and a decreasing function of the search cost parameter (i.e. the cost of drawing one worker from the labor force),  $C$ . Mechanistically, of course, both points are straightforward. A producer's optimal size is determined by the level of  $N$  that maximizes the difference between two pieces: (i) the expected gross payoff net of the expected turnover cost and (ii) the coordination cost. Given that the first piece follows as

$$N(\alpha k^\alpha q_1^{1-\alpha} - \frac{C\delta}{\mu})$$

for a firm with cutoff  $q_1$ , and

$$N(\alpha k^\alpha (q_1^{1-\alpha} \mu + q_2^{1-\alpha} (1 - \mu)) - C\delta)$$

for a firm with cutoff  $q_2$ , it is apparent that both terms increase as either  $C$  falls or  $\mu$  rises (recall,  $q_1 > q_2$ ). A drop in  $C$ , naturally, makes the replacement of a worker less costly for any producer. A higher value of  $\mu$  increases the net expected gross payoff of a firm with cutoff  $q_1$  by making it easier to locate a high-skill worker. For a firm with cutoff  $q_2$ , a rise in  $\mu$  increases the net expected gross payoff by increasing the average fraction of high-skill workers the producer employs over time.

More importantly, the absolute amount by which these two expressions increase is itself an increasing function of  $N$ . Thus, because the coordination cost schedule,  $N^\phi$ , does not

change with  $C$  or  $\mu$ , the firm's problem will be satisfied by a larger value of  $N$  as either  $C$  falls or  $\mu$  rises.

This first result, I should note, is somewhat limited in that it derives the effects of changes in  $C$  and  $\mu$  on  $N(q_1)$  and  $N(q_2)$  under the assumption that a producer's cutoff skill level remains constant. A more general result linking a producer's optimal scale to a change in either of these quantities appears below. First, I establish the following property characterizing the relationship between reservation skill levels and firm sizes.

**Proposition 3:** The firm maximizes its expected profit, selecting a cutoff of  $q_1$  and size  $N(q_1)$  rather than  $q_2$  and  $N(q_2)$  if and only if  $N(q_1) \geq N(q_2)$ .

*Proof:* Using the expressions for expected profits,  $\pi(q_1) \geq \pi(q_2)$  implies the following:

$$(\alpha k^\alpha q_1^{1-\alpha} - \frac{C\delta}{\mu} - N(q_1)^{\phi-1})N(q_1) \geq (\alpha k^\alpha (q_1^{1-\alpha}\mu + q_2^{1-\alpha}(1-\mu)) - C\delta - N(q_2)^{\phi-1})N(q_2).$$

Substituting the expressions for  $N(q_1)$  and  $N(q_2)$  into the above equation,  $\pi(q_1) \geq \pi(q_2)$  implies that

$$(\alpha k^\alpha q_1^{1-\alpha} - \frac{C\delta}{\mu})^{\frac{\phi}{\phi-1}} \geq (\alpha k^\alpha (q_1^{1-\alpha}\mu + q_2^{1-\alpha}(1-\mu)) - C\delta)^{\frac{\phi}{\phi-1}}$$

which, because  $\frac{\phi}{\phi-1} > 0$ , can be further simplified to

$$\alpha k^\alpha (q_1^{1-\alpha} - q_2^{1-\alpha}) \geq \frac{C\delta}{\mu}.$$

This above expression is equivalent to the one that ensures that  $N(q_1) \geq N(q_2)$ , which can be found by simplifying

$$(\alpha k^\alpha q_1^{1-\alpha} - \frac{C\delta}{\mu})^{\frac{1}{\phi-1}} \geq (\alpha k^\alpha (q_1^{1-\alpha}\mu + q_2^{1-\alpha}(1-\mu)) - C\delta)^{\frac{1}{\phi-1}}.$$

Hence,  $\pi(q_1) \geq \pi(q_2) \iff N(q_1) \geq N(q_2)$ .

■

Given Propositions 2 and 3, I now state the following key result.

**Proposition 4:** An increase in the proportion of type 1 workers,  $\mu$ , and/or a decrease in the search cost parameter,  $C$ , will increase the number of workers that a producer optimally hires.

*Proof:* There are four cases to consider: two in which changes in  $\mu$  and/or  $C$  leave a firm's initial reservation skill unchanged and two in which the firm's cutoff switches. In either of the first two cases where the cutoff does not change, Proposition 2 proves the result.

Suppose that a firm has initial cutoff equal to  $q_1$  so that  $N(q_1) > N(q_2)$  by Proposition 3. Now let either  $\mu$  increase and/or  $C$  decrease. From Proposition 2, the resulting values of  $N(q_1)$  and  $N(q_2)$ , denoted  $N(q_1)'$  and  $N(q_2)'$ , must both increase. Again, from Proposition 3, if the firm's new cutoff is  $q_2$ , then it follows that  $N(q_2)' > N(q_1)' > N(q_1) > N(q_2)$ . Likewise, if the firm's initial cutoff is  $q_2$  but subsequent cutoff is  $q_1$ , then it follows that  $N(q_1)' > N(q_2)' > N(q_2) > N(q_1)$ .

■

Thus, a drop in the cost associated with drawing a worker from the labor pool or an increase in the proportion of high-skill workers in the labor force will increase a producer's optimal size, regardless of its impact on the producer's cutoff level of skill.

Notice, this simple framework can account for the evidence documented by Barron et

al. regarding the relationship between employer size and recruiting effort. Consider, for example, two markets that differ only with respect to the search cost parameter,  $C$ . If the firm in the market with the lower cost (firm 1) and, thus, greater optimal employment, has a cutoff of  $q_1$ , while the firm in the market with the higher cost (firm 2) has a cutoff of  $q_2$ , firm 1 will, on average, review more candidates for each job opening than firm 2.<sup>8</sup> Hence, if large producers are more selective than small producers with respect to the workers they are willing to hire, large firms will search more extensively than small firms when filling a vacancy.

### 2.3 Type-Specific Turnover Rates

Although the model above restricts the separation rate,  $\delta$ , to be uniform across all workers, this assumption can be relaxed with little change in the results. Suppose that workers have type-specific separation rates:  $\delta_1, \delta_2, \dots, \delta_T$  that are (weakly) decreasing in skill, so that  $\delta_1 \leq \delta_2 \leq \dots \leq \delta_T$ . This parameter restriction is compatible with a host of empirical evidence (e.g. Pencavel 1970, Parsons 1972, and Neal 1998) that has established an inverse relationship between rates of turnover and wages.<sup>9</sup>

In such a case, a firm with a cutoff equal to  $q_p$  will now face the following problem:

---

<sup>8</sup>If a firm initially has cutoff equal to  $q_2$ , a drop in  $C$  may induce the firm to adopt a cutoff of  $q_1$  subsequently. Thus, this particular scenario is certainly possible. The reverse, as it turns out, is not true. A firm with a cutoff of  $q_1$  will not switch to a cutoff of  $q_2$  in response to a drop in  $C$  because  $|\frac{\partial \pi(q_1)}{\partial C}| > |\frac{\partial \pi(q_2)}{\partial C}|$  when  $N(q_1) > \mu N(q_2)$  which will hold if  $q_1$  is initially the cutoff (recall Proposition 3).

<sup>9</sup>This restriction also preserves the producer's use of a cutoff strategy. Note that if more highly skilled workers have *higher* rates of turnover, a firm that is willing to hire a worker with skill  $q_p$  would not necessarily be willing to hire a worker of greater skill. Although doing so would certainly increase the firm's expected gross payoff, it would not necessarily decrease the expected turnover cost (see below).

$$\max_N \alpha k^\alpha N \left( \sum_{j=1}^p \frac{q_j^{1-\alpha} \mu_j}{\mu_1 + \mu_2 + \dots + \mu_p} \right) - \frac{CN \sum_{j=1}^p \delta_j \mu_j}{(\mu_1 + \mu_2 + \dots + \mu_p)^2} - N^\phi. \quad (11)$$

The only difference between this objective function and that given by (7) lies in the specification of the expected turnover cost. To be sure, it still follows as the product of two terms: the expected cost per replacement and the average number of replacements that must be found each period. The first term is simply the same as before: with a cutoff of  $q_p$ ,  $\frac{C}{\mu_1 + \mu_2 + \dots + \mu_p}$  units of output are required, on average, to fill a vacated position. The average number of replacements, given by the product of  $N$  and the average turnover rate, now follows as

$$\frac{N \sum_{j=1}^p \delta_j \mu_j}{\mu_1 + \mu_2 + \dots + \mu_p}.$$

Solving (11) yields the following expression for a firm's optimal size

$$N(q_p) = \left( \frac{\alpha k^\alpha \sum_{j=1}^p q_j^{1-\alpha} \mu_j}{\phi(\mu_1 + \mu_2 + \dots + \mu_p)} - \frac{C \sum_{j=1}^p \delta_j \mu_j}{\phi(\mu_1 + \mu_2 + \dots + \mu_p)^2} \right)^{\frac{1}{\phi-1}} \quad (12)$$

which, for the two-type case, implies an optimal employment of

$$N(q_1) = \left( \frac{\alpha k^\alpha q_1^{1-\alpha}}{\phi} - \frac{C \delta_1}{\phi \mu} \right)^{\frac{1}{\phi-1}} \quad (13)$$

for a cutoff of  $q_1$ , and

$$N(q_2) = \left( \frac{\alpha k^\alpha (q_1^{1-\alpha} \mu + q_2^{1-\alpha} (1-\mu)) - C(\delta_1 \mu + \delta_2 (1-\mu))}{\phi} \right)^{\frac{1}{\phi-1}} \quad (14)$$

for a cutoff of  $q_2$ . Notice that from this more general framework, the implications regarding firm size with respect to changes in the cost of drawing a worker from the labor force,  $C$ , still hold. All else constant, a lower value of  $C$  induces a producer to hire more workers regardless of the impact on the producer's cutoff.

It also follows in this more general formulation that an increase in the proportion of high-skill workers,  $\mu$ , will lead to the producer choosing to operate with greater numbers of workers. The rationale for this result ought to be apparent. For firms with a cutoff of  $q_1$ , an increase in  $\mu$  reduces the expected turnover cost relative to the expected gross payoff by making type 1 workers easier to find. For firms with a cutoff of  $q_2$ , a higher value of  $\mu$  increases the expected gross payoff by increasing the average fraction of high-skill workers who are hired and, simultaneously, decreases the expected turnover cost by increasing the average fraction of 'low-turnover' workers hired over time.

This latter point, interestingly, suggests that as the proportion of high-skill workers in the labor force rises, firms not only become larger, but also may exhibit lower average rates of turnover. Note that if a firm has a cutoff of  $q_1$ , the turnover rate remains equal to  $\delta_1$ . As described above, however, if a firm has a cutoff of  $q_2$ , the average turnover rate,  $\delta_1\mu + \delta_2(1 - \mu)$ , decreases. In a cross section of producers consisting of both types, then, there may be an inverse relationship between average plant size and turnover that would be consistent with Davis et al.'s evidence on job flows cited in the Introduction.

### 3 Concluding Discussion

The model developed above has two primary implications. First, firms will tend toward larger sizes in markets in which a greater proportion of the overall labor force is skilled. This implication, of course, has a long tradition in technological theories of the firm (e.g. Lucas 1978, Rosen 1982, and Kremer 1993) that suggest that larger organizations require more capable individuals. More competent managers, for instance, might be able to monitor and coordinate greater numbers of workers more effectively than less competent ones. Alternatively, large firms may utilize a more extensive division of labor that places greater importance on the successful completion of any particular task. Such organizations, therefore, will rely on more skilled individuals performing those tasks.

The framework offered here merely suggests that a larger fraction of high-skill workers in the labor force increases the average level of skill among the workers that a firm hires over time. Because physical capital and worker skill are complementary in the model, the expected return to hiring an additional worker increases with the average skill level of the labor force. The result is a larger optimal firm size.

Second, and more importantly, the model implies that producers will operate at a larger scale when the cost of replacing workers is smaller. Although various labor market institutions such as employment agencies may facilitate the firm-working matching process, the literature studying urban labor markets has suggested that the presence of a large number of workers who possess skills specific to a particular industry will offer firms within that industry an environment in which worker replacement is relatively easy.

To be sure, the connection between geographic agglomeration and productivity gains has been the subject of considerable research. Localization economies and their effects

on industrial growth, for example, have been quantified by numerous studies including Henderson (1986), Glaeser et al. (1992), and Henderson et al. (1995). Although firms may benefit from industrial concentration for a variety of reasons, Henderson (pp. 47-48) suggests that localization economies may, at least in part, be the result of “labor market economies where industry size reduces search costs for firms looking for workers with specific training relevant to that industry.”<sup>10</sup>

Why might search costs be lower when the extent of the local (relevant) labor market is greater? One possibility, certainly, is that information may flow more extensively in populous areas so that a given job advertisement, for example, may be seen by a larger number of potential employees. Hence, paying a certain cost to recruit an employee for a job opening may produce a larger number of applicants in a thick market than in a thin one.<sup>11</sup>

Although little empirical work has examined firm and worker search patterns in local geographic areas, there is some evidence to support this idea. Dumais et al. (1997), for instance, find that the decision of a firm to locate in a metropolitan area is strongly tied to the presence of workers whose occupations match well with that firm’s needs. Such a result, of course, is compatible with the idea that large pools of ‘appropriate’ workers facilitate the search for employees and, thus, tend to attract producers. Additionally, in a study of urban markets in Israel, Alperovich (1993) finds that unemployment duration is significantly lower in more populous areas. This result may indicate that firms and workers are able to conduct

---

<sup>10</sup>This idea actually dates back to Marshall (1920) who suggested that labor market search considerations are an important reason for localization.

<sup>11</sup>Hence, for a given turnover rate  $\delta$ , dense urban markets may reduce  $C$  by increasing the number of ‘contacts’ or ‘interactions’ that occur between agents.

relatively extensive searches more easily and, thus, find acceptable matches more quickly in thicker markets.

In terms of the model presented above, this idea can be captured as follows. Suppose that paying a fixed search cost,  $F$ , attracts a flow of  $\lambda$  candidates. The per worker search cost  $C$  (i.e. the cost of taking a single draw from the available pool of labor) is then  $\frac{F}{\lambda}$ . If, as just suggested,  $\lambda$  increases with the extent of the relevant labor market,  $C$  should be lower in markets with larger supplies of appropriate labor. Following Proposition 4, thick markets should produce larger production establishments.

Interestingly, such an implication appears to have some empirical support. Holmes and Stevens (2002), for instance, find that production establishments located in areas (e.g. Census divisions, metropolitan areas, and counties) where an industry is concentrated tend to be larger than establishments in areas where an industry is less heavily represented. Henderson (p. 58) finds a similar result, noting that city-industry employment and average plant size are positively correlated in U.S. and Brazilian manufacturing data.

One intriguing conjecture that emerges from these results, then, concerns the nature of localization economies. Although many explanations have been suggested for the positive link between spatial agglomeration and productivity gains within industries, part of those gains may be associated with producers operating on a larger scale. As noted in the Introduction, empirical evidence indicates that larger establishments pay higher wages (Brown and Medoff 1989, Troske 1999) and enhance worker productivity (Idson and Oi 1999). The extent to which these two well-established empirical regularities, localization economies and the producer-size wage premium, are connected, therefore, may be a promising avenue for future research.

## Appendix

**Table 1: Establishment Size and Search Activity Per Job**

Establishment Size (Employees)	Hours Spent Recruiting, Screening, and Interviewing	Number of Applicants Interviewed
1 to 9	6.17	5.19
10 to 25	7.14	6.27
26 to 250	9.35	6.97
251 to 4715	12.74	8.26

Note: Results reported by Barron et al. (1985, p. 46).

## References

- Alperovich, G., 1993. City size and the rate and duration of unemployment: evidence from Israeli data. *Journal of Urban Economics* 34, 347-357.
- Audretsch, D., Mahmood, T., 1995. New firm survival: new results using a hazard function. *Review of Economics and Statistics* 77, 97-103.
- Barron, J., Bishop, J., Dunkelberg, W., 1985. Employer search: the interviewing and hiring of new employees. *Review of Economics and Statistics* 67, 43-52.
- Brown, C., Medoff, J., 1989. The employer size-wage effect. *Journal of Political Economy* 97, 1027-1059.
- Cinlar, E., 1975. *Introduction to stochastic processes*. Englewood Cliffs, NJ: Prentice-Hall.
- Coase, R., 1937. The nature of the firm. *Economica* 4, 386-405.
- Cohen, W., Levin, R., 1989. Empirical studies of innovation and market structure. In: R. Schmalensee and R. Willig (Eds.). *Handbook of Industrial Organization*, Vol. II. New York: North-Holland, pp. 1059-1108.
- Davis, S., Haltiwanger, J., Schuh, S., 1996. *Job creation and destruction*. Cambridge, MA: MIT Press.
- Dumais, G., Ellison, G., Glaeser, E., 1997. Geographic concentration as a dynamic process. NBER Working Paper No. 6270.
- Dunne, T., Roberts, M., Samuelson, L., 1989. The growth and failure of U.S. manufacturing plants. *Quarterly Journal of Economics* 104, 671-698.
- Evans, D. S., 1987. Tests of alternative theories of firm growth. *Journal of Political Economy* 95, 657-674.
- Fukuyama, F., 1995. *Trust: the social virtues and the creation of prosperity*. New York: Free Press.
- Glaeser, E., Kallal, H., Scheinkman, J., Shleifer, A., 1992. Growth in cities. *Journal of Political Economy* 100, 1126-1152.
- Grossman, S., Hart, O., 1986. The costs and the benefits of ownership: a theory of vertical and lateral integration. *Journal of Political Economy* 94, 691-719.
- Hart, O., Moore, J., 1990. Property rights and the nature of the firm. *Journal of Political Economy* 98, 1119-1158.
- Henderson, V., 1986. The efficiency of resource usage and city size. *Journal of Urban Economics* 19, 47-70.
- Henderson, V., Kuncoro, A., Turner, M., 1995. Industrial development in cities. *Journal of Political Economy* 103, 1067-1090.

- Holmes, T., Stevens, J., 2002. Geographic concentration and establishment scale. *Review of Economics and Statistics* 84, 682-690.
- Idson, T., Oi, W., 1999. Workers are more productive in large firms. *American Economic Review* 89, 104-108.
- Kremer, M., 1993. The o-ring theory of economic development. *Quarterly Journal of Economics* 108, 551-575.
- Kumar, K., Rajan, R., Zingales, L., 1999. What determines firm size? NBER Working Paper No. 7208.
- Lucas, R. E., 1978. On the size distribution of business firms. *Bell Journal of Economics* 9, 508-523.
- Marshall, A., 1920. *Principles of economics*. London: MacMillan.
- Neal, D., 1998. The link between ability and specialization. *Journal of Human Resources* 33, 173-200.
- Oi, W., 1962. Labor as a quasi-fixed factor. *Journal of Political Economy* 70, 538-555.
- Parsons, D., 1972. Specific human capital: an application to quit rates and layoff rates. *Journal of Political Economy* 80, 1120-1143.
- Pencavel, J., 1970. *An analysis of the quit rate in American manufacturing*. Princeton: Princeton University Press.
- Rosen, S., 1982. Authority, control, and the distribution of earnings. *Bell Journal of Economics* 13, 311-323.
- Troske, K., 1999. Evidence on the employer size-wage premium from worker-establishment matched data. *Review of Economics and Statistics* 81, 15-26.
- Waldman, M., 1984. Worker allocation, hierarchies, and the wage distribution. *Review of Economic Studies* 51, 95-109.
- Williamson, O. E., 1967. Hierarchical control and optimum firm size. *Journal of Political Economy* 75, 123-138.